Resonant Tunnelling Spectroscopy of the Local Density of States in a Disordered Conductor

A. K. Savchenko (a), J. P. Holder (a), V. I. Falko (b), B. Jouault (c), G. Faini (c), F. Laruelle (c), and E. Bedel (d)

(a) Department of Physics, University of Exeter, Stocker Road, Exeter, EX4 4QL, UK
(b) School of Physics and Chemistry, Lancaster University, Lancaster, LA14YB, UK
(c) L2M-CNRS, 196 Avenue H. Ravera, B.P. 107, F-92225 Bagneux, Cedex, France
(d) LAAS-CNRS, F-31077 Toulouse Cedex, France

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We have studied the effects of magnetic field on the fluctuations of the electron density of states in a disordered conductor, a contact in the resonant tunnelling structure GaAlAs/GaAs/GaAlAs. The local density of states (LDOS) in the contact is detected by an impurity which acts as a 'spectrometer'. In the regime of weak magnetic field, $\omega_c \tau \ll 1$, a linear increase of the fluctuation magnitude with $B^2$ is seen, due to the decrease of the volume where the LDOS is formed and a decrease in the statistical averaging. In the regime $\omega_c \tau \gg 1$, large oscillations of the fluctuation magnitude have been observed. We suggest an explanation of this new effect, de Haas-van Alphen type oscillations in the fluctuation magnitude. The behaviour of impurity spectrometers in a magnetic field and images given by a complex spectrometer are also discussed.

Introduction  Resonant tunneling (RT) through individual impurities in vertical mesoscopic structures has been studied in a series of experiments [1 to 4]. An impurity manifests itself as a step-like feature in $I(V)$ which occurs when its level is aligned, with increasing source–drain bias $V_{sd}$, with the Fermi level in the contact. In this paper, we describe how such resonant tunnelling is used for the spectroscopy of the contacts. The current through the impurity is determined by the transmission probability $T(E)$ as well as the density of states in the contact $\nu(E)$. If the barrier is high, it is not changed significantly over a small range of $V_{sd}$, the current becomes a measure of the single-particle density of electron states in the contacts, at the impurity energy $E_S$.

$$I(V) \propto \nu(E_S).$$  \hspace{1cm} (1)

The spectrum $\nu(E)$ below the Fermi level can then be measured. It is important that in this method, with a localised state acting as a spectrometer, information about the local density of states, LDOS, is obtained at the point $r$ in the contact from where the tunnelling occurs. The onset of the step in $I(V)$ is smeared due to the thermal distribution of carriers in the contact, while the current above the threshold shows temperature independent features [2, 3] which have been attributed to the fluctuating LDOS [4].

There have been a number of theoretical studies [5,6] of the LDOS fluctuations in the metallic regime, $k_F l \gg 1$, although few experiments have been performed. Direct
measuring of the LDOS allows one to get unique information about mesoscopic conductors which is not available from widely studied conductance fluctuations. The aim of this work is to study how fluctuations of the LDOS in a 3D disordered metal, doped GaAs, vary with magnetic field. To resolve small-scale fluctuations and to suppress the slowly varying background in $I(V)$, the differential conductance $G(V) = dI/dV(V)$ is measured and its variance $\langle (\delta G)^2 \rangle = \langle (G - G_{av})^2 \rangle V$ is analysed as a function of $B$.

**Experimental Structure** The structure consists of a 50 Å GaAs well embedded between two 81 Å Al$_{0.33}$Ga$_{0.67}$As barriers. The top and bottom contacts, fabricated from $2 \times 10^{17}$cm$^{-3}$ Si doped GaAs, are separated from the barriers by undoped GaAs spacers of 300 and 200 Å, respectively. The conducting area of the structure in the plane is reduced to a 700 Å diameter disk by ion bombardment [8]. This is done to decrease the number of conducting impurities and avoid the overlap of the LDOS spectra. A schematic band diagram of the device with an impurity S is shown in Fig. 1. We selected a sample with a distinct impurity level, which is well separated in the energy scale from other localised states and states in the quantum well, this “clear” energy interval of about 10 mV determines the energy range of the spectrometer.

**Characteristics of the Spectrometer** The spectrometer probe has a finite width, $\Gamma \approx \Gamma_{\text{max}}$, determined by the coupling between the impurity and the contacts. It appears that the top barrier is effectively thinner (due to the bombardment) and for a positive bias is also cut by electric field. Thus, it is this (collector) barrier that determines $\Gamma_{\text{max}}$. The coupling $\Gamma_{\text{min}}$ is determined by the less transparent emitter barrier. In the differential conductance the threshold in current corresponds to a peak with width $\beta \Gamma/e$, where $\Gamma \approx 120$ μeV and the factor $\beta = dE/d(eV) = 0.24$ relates the voltage and energy scales and was found from the temperature smearing of the peak. The current above the threshold is determined by the lowest leak rate, $I = e\Gamma_{\text{min}}/h$, and thus carries information about the LDOS in

Fig. 1. Band diagram of the resonant tunneling structure with a spectrometer (top). Differential conductance as a function of bias and energy, with the threshold peak and the ‘fingerprint’ of the LDOS below the Fermi level in the emitter.
the emitter. Figure 2 shows an example of \( I(V) \) where one can see a temperature dependent threshold followed by a temperature independent ‘fingerprint’ of the fluctuating LDOS in the emitter.

**Fluctuation Magnitude of the LDOS Measured by the Resonant Tunnelling Spectrometer** The features in \( G(V) \) are a reflection of the fluctuating density of single-electron wave functions, \( |\psi_j(r)|^2 \) [9]. The magnitude of these fluctuations, with correlation energy \( \Gamma \), is determined by the correlations between energetically close eigenstates [9, 10]. For a random difference between two values of \( n \) in the neighbouring \( \Gamma \)-intervals their variance in 3D [7] is

\[
\langle \delta n^2 \rangle \sim n \Gamma^{-1} L_F^{-3},
\]

where \( L_F = \sqrt{Dh/\Gamma} \) is the Thouless length. A simple way to visualize this result, that it is the volume \( L_F^3 \) of the contact which contributes to the measured fluctuations, is to think of the diffusive motion of electrons for a ‘waiting’ time of \( h/\Gamma \) before tunnelling, Fig. 3a. One can then estimate the correlation magnetic field of the fluctuations (at zero magnetic field) as \( \Delta B_c \approx \Phi_0/L_F^2 \), where \( \Phi_0 \) is the flux quantum.

The conductance \( G = dI/dV \) is a measure of the derivative, with respect to energy, of the LDOS. Thus, the variance \( \langle \delta G^2 \rangle \) can be calculated from the variance in \( n \) as \( \langle \delta G^2 \rangle \propto \langle \delta n^2 \rangle / V_F^2 \), where \( V_F = \Gamma/\beta e \) is the correlation voltage. As the fluctuations are proportional to the threshold peak \( G_F \), the variance \( \langle \delta G^2 \rangle \) can be normalised by its height and expressed for a system with an anisotropic diffusion tensor \((D_x, D_y, D_z)\) as

\[
\langle \delta G^2 \rangle / G_F^2 \approx (\Gamma/hD)^{1/2}/(\nu h D) = h^{-3/2} \Gamma^{1/2} v^{-1} (D_x D_y D_z)^{-1/2}.
\]

In contrast to the universal conductance fluctuations in metals, the fluctuations of the tunneling conductance (3) depend on the diffusion coefficient \( D \) as well as the spectrometer width \( \Gamma \). By measuring the differential conductance we emphasise those fluctuations which are determined by scale \( L_F \) and ignore the fluctuations on a scale of the mean free path \( l \). The latter are large in magnitude but also have large correlation energy, \( h/\tau > E_F > \Gamma \), so that they cannot be resolved in the studied energy range.

Magnetic field changes the characteristic volume \( L_F^3 \) and thus affects the fluctuation magnitude. The cyclotron motion in the presence of a classically small magnetic field, \( \omega_c \tau \leq 1 \), in the \( z \)-direction suppresses transverse diffusion, so that
$D_{x,y} = D/(1 + (\omega_c t)^2)$. Then the fluctuations should increase [7], Fig. 3a, as

$$\frac{\langle \delta G^2 \rangle_B}{\langle \delta G^2 \rangle_{B=0}} \approx 1 + (\omega_c t)^2.$$  \hspace{1cm} (4)

It was our first aim to see this effect of magnetic field on the fluctuation magnitude.

**Some General Aspects of the LDOS Spectroscopy in Magnetic Field**

1. There is a diamagnetic shift of the spectrometer level which shifts the threshold peak, together with the images of the LDOS, in the positive $V_{sd}$ direction.

2. In quantising magnetic fields, oscillations of the Fermi energy $E_F(B)$ lead to oscillations in the position of the threshold peak. When only the lowest spin-degenerate Landau band is left occupied, the Fermi level starts rising rapidly with $B$ (magnetic freeze-out).

3. At large fields, the Zeeman splitting of the impurity level will produce a spin-sensitive spectrometer.

**Images of the LDOS in Magnetic Field: General View**

Figure 4 shows, in a grey-scale, the dependence $G(V)$ measured in magnetic field between 0 and 15 T, applied parallel to the current and changed with a step of 20 mT. The curves are offset upwards and multiplied by an increasing with $B$ factor in order to compensate for suppression of the tunneling current. Here are some conclusions drawn from the overall view of the picture:
a) From the diamagnetic shift of the threshold peak, Fig. 5a, the localisation radius of the spectrometer is calculated as \( R \approx 67 \) Å, on the basis of a simple parabolic model with confining energy \( \hbar \omega_0 \approx 45 \text{ meV} \) used as adjustable parameter,

\[
E_S(\omega_0, B) = \left( (\hbar \omega_0)^2 + (\hbar \omega_c)^2 \right)^{1/2}, \tag{5}
\]

\[
\hbar \omega_0(R) = \omega_c^2 m^* R^2 / 2. \tag{6}
\]

b) Oscillations of the Fermi level in the emitter are seen, with magnetic freeze-out occurring at \( B \approx 9.5 \) T. From this field the electron concentration is estimated as \( n \approx 2.7 \times 10^{17} \text{ cm}^{-3} \) (using the relation \( n = \left( 2/\pi^2 \right)^{1/2} (eB/\hbar)^{3/2} \)), which is close to the nominal doping of the electrodes.

c) At \( B \leq 3 \) T the fluctuating pattern in \( G(V, B) \) changes randomly with varying magnetic field, with a correlation magnetic field in the region \( B \approx 0 \) of the order \( \Delta B_c \)

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Fig. 4. Grey-scale image of the conductance fluctuations (offset upwards with increasing magnetic field), brighter image corresponds to larger conductance. The peak at 95 mV is due to the tunneling through a confined state in the quantum well. Inset: a spectrogram with many impurities seen at negative bias.
At higher fields, however, the fluctuations transform into a more regular pattern where individual features of Landau bands tend to move towards the Fermi level as $E_n^L(B) = \hbar \omega_c (n + 1/2)$.

d) At fields above 6 T the Zeeman splitting of the threshold peak is seen, corresponding to two spin states crossing the Fermi level, with a $g$-factor of about 0.45, Fig. 6. This value is close to that in bulk GaAs and differs from $g \approx 0.22$ seen in [11] for an impurity in a 50 Å quantum well. This suggests that our spectrometer is positioned away from the centre of the quantum well, and pos-
sibly in one of the barriers. The spin-resolved images of the Landau bands depend on the relation between the \( g \)-factors of the impurity and the electrode. If the spectrometer splitting is smaller than that of a Landau band, the two images do not coincide and are shifted along \( V_{sd} \). When the splitting is equal, the two images coincide once both spectrometer states are below the Fermi level. In Fig. 5a, b one can trace local images of a Landau band, with only one spin-state detected (after the first threshold peak), that are not shifted but simply amplified after the second threshold peak, when the image of the other spin state is added.

In this figure an example of a complex spectrometer with two states at zero field is shown, probably originating from two closely positioned donors (a ‘donor molecule’). As the two levels give the same set of images but shifted along \( V_{sd} \), Fig. 5, the distance between the two impurities \( r_{ij} \ll L_f \). In some cool-downs of the sample, with a slightly different position of the impurities, the second level appears to be weaker, which could result from the Coulomb blockade of the current through the two states [11, 12].

An interesting effect is seen in strong magnetic fields. The features of the Landau band are bent when approaching the Fermi level, indicating some weakening of the \( B \) dependence of the Landau band. We suggest that this implies an increase of the interactions between electrons near the Fermi level, although we cannot propose a clear explanation of this effect.

e) In the region of negative bias a dense set of impurities is seen in the spectrogram, Fig. 4, inset, which are those that were positioned at \( V_{sd} = 0 \) below the Fermi level in the emitter. Because of the small energy range between these levels and a rapid increase of the background current, it is not possible to perform the LDOS spectroscopy for negative bias.

**Fluctuation Magnitude of the LDOS in Weak Fields** For the statistical analysis a simple sample realisation was chosen, with a single level dominating the spectrogram, Fig. 7. A range of fields below 10.5 T was chosen to neglect the spin splitting effects. The variance \( (\delta G)^2 = \langle (G(B,V) - \langle G(B,V) \rangle)^2 \rangle \) is calculated for each magnetic field (the brackets \( \langle \rangle \) denote averaging over the range \( \Delta V \approx 6.3 \text{ mV} \) after the threshold peak over \( \approx 1.5 \text{ meV} \) below the Fermi level) and its value is normalised by the value \( G_{ri} \). To reduce the scatter, a further averaging of the results over a \( B \) range of 0.25 T is then performed for the range of \( B \leq 4 \text{ T} \). The result is presented in Fig. 3b, where an increase in the fluctuations is seen, which is in reasonable agreement with the expected quadratic dependence Eq. (3). From this dependence the value of the momentum relaxation time, \( \tau \approx 9 \times 10^{-14} \text{ s} \), and the mobility, \( \mu = 0.22 m^2/Vs \), are calculated, and agree with those expected for the emitter with a given doping concentration [13].

These values give \( B \approx 4 \text{ T} \) for the crossover \( \omega_c \tau \approx 1 \), which corresponds with the range of fields where the formation of Landau bands is seen. This mobility justifies our use of the diffusion approximation (since \( \tau \Gamma / \hbar \approx 10^{-2} \)), and allows us to estimate the diffusion coefficient \( D(B = 0) \approx 40 \text{ cm}^2/Vs \), using \( E_F = 18 \text{ meV} \) for the nominal doping.

The expected correlation magnetic field \( \Delta B_c \approx \Gamma / eD \approx 0.03 \text{ T} \) gives a correct order of magnitude consistent with the experimental value (step 0.02 T does not allow the comparison \( \Delta B_c \) values more accurately). At zero field the correlation voltage is calculated as \( \Delta V_c \approx 0.5 \text{ mV} \), which is close to the spectrometer width \( \Gamma \). At the same time, an increase of the correlation voltage is seen at \( V_{sd} > 0.056 \text{ mV} \), Fig. 7, which is an
indication that the life-time of the quasi-particles which are well below the Fermi level decreases due to electron–electron interaction, so that an additional energy broadening is added to the spectrometer width $G$. 

Fluctuation Magnitude of the LDOS in Strong Magnetic Fields  In the $\omega_c \tau \geq 1$ regime the fluctuation magnitude has shown a strikingly different dependence, a series of well defined peaks which are periodic in $1/B$, Fig. 8. It is important to note that these oscillations are much stronger than those in $G_T(B)$, which are due to the modulation of the density of states caused by the depopulation of the Landau bands, Fig. 8b. The ob-
served oscillations are also larger than the Shubnikov-de Haas (SdH) oscillations, Fig. 8, inset, measured on a lateral structure with the same doping.

To explain the origin of these oscillations let us refer back to Eq. (3). In the weak magnetic field regime, there was a decrease of the perpendicular diffusion coefficients $D_x, D_y$ which caused an increase in the fluctuations. In strong fields, the perpendicular motion of electrons is suppressed and becomes quasi-one-dimensional, with the diffusion coefficient along the field $D_z = (V_z^F)^2 \tau$ dependent on $B$. At a given field, $D_z$ is smallest in the highest filled Landau band, where $V_z^F$ and $\tau$ are small. The decrease in $D_z$ will occur periodically every time a band minimum approaches the Fermi level. In these oscillations there are two contributing factors: oscillations in $V_z^F$ and in $\tau$. To explain why oscillations in the LDOS fluctuations are stronger than $G_T(B)$ and SdH oscillations, we assume that the inter-Landau band scattering is suppressed near the minimum of the Landau band, when the length scale $L_z^F = \sqrt{\hbar D_z^{(n)}/\Gamma}$ becomes smaller.
than the interband scattering. The states in the highest filled band then play the dominant role in the total fluctuation magnitude $\langle \delta G^2 \rangle$ [9]. These states provide an enhanced contribution to the LDOS fluctuations compared to the typical variance $\langle \delta^{(\text{typ})} G^2 \rangle$.

$$\langle \delta^{(n)} G^2 \rangle / \langle \delta^{(\text{typ})} G^2 \rangle \approx \left( \nu^{(n)} / \nu \right) \left( D_0 / D_z^{(n)} \right)^{1/2}. \tag{7}$$

The oscillations in the SdH effect are determined by an increase of the density of states, so that their magnitude is represented by the first factor in (7). It is the second, additional factor $\left( D_0 / D_z^{(n)} \right)^{1/2}$ which is responsible for the enhancement of the oscillations in our case. Its existence is a specific feature of the LDOS fluctuations.

**Conclusion** In summary, we have performed a statistical analysis of the images of the LDOS in a disordered 3D conductor. We have seen a quadratic increase of the fluctuation magnitude at small magnetic fields which is followed by enhanced oscillations of the fluctuation magnitude at strong fields. We attribute these effects to the behaviour of the characteristic volume in the emitter where the LDOS fluctuations are formed.

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**References**